BLUE VALLEY DISTRICT CURRICULUM

MATHEMATICS

Pre-Calculus &
Honors Pre-Calculus

	ORGANIZING THEME/TOPIC	CONTENT (PH CHAPTER REFERENCE)	FOCUS STANDARDS & SKILLS
1-2	Prerequisite Algebra	*Real Numbers [P.1] *Cartesian Coordinate System [P.2] *Linear Equations and Inequalities [P.3] *Lines in the plane [P.4] *Solving equations graphically, numerically, and algebraically [P.5] *Complex numbers [P.6] *Solving Inequalities algebraically and graphically [P.7] *Supplement other topics as necessary *Some schools may handle some of these topics through a Review Assignment	Reasoning with Equations and Inequalities A-REI **Understand solving equations as a process of reasoning and explain the reasoning.** 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **Solve equations and inequalities in one variable.** 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. 4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. Expressing Geometric Properties with Equations G-GPE **Translate between the geometric description and the equation for a conic section.** 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. Creating Equations★ A-CED **Create equations that describe numbers or relationships.** 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* The Complex Number System N-CN **Perform arithmetic operations with complex numbers.** 1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. 2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. 3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **Use complex numbers in polynomial identities and equations.** 4. Solve quadratic equations with real coefficients that have complex solutions.

4	Functions and Graphs	*Modeling and equation solving (1.1) *Functions and their properties (1.2) *Twelve Basic Functions (1.3) *Building functions from functions (1.4) * Inverses (1.5) *Graphical transformations (1.6) *Modeling with functions (1.7) *Defining a function piecewise including rewriting absolute value functions as a piecewise function.	Interpreting Functions F-IF **Understand the concept of a function and use function notation.** 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Interpret functions that arise in applications in terms of the context.** 3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ 4. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **Analyze functions using different representations.** 5. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. 6. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression* Building Functions F-BF **Build a function that models a relationship between two quantities.** 1. Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* **Build new functions from existing functions.** 2. Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* 3. Find inverse functions. a. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1.* b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

5-6	Polynomial, Power, and Rational Functions	*Linear and Quadratic Functions and Modeling (2.1) (Average Rate of Change & Difference Quotient and investigate concavity as a rate of change) *Power Functions with Modeling (2.2)- *HONORS* *Polynomial Functions of Higher Degree with Modeling (2.3) *Real Zeros of Polynomial Functions (2.4) *Complex Zeros and the Fundamental Theorem of Algebra (2.5) *Graphs of Rational Functions (2.6) *Solving Equations in 1 variable (2.7) *Solving Inequalities in 1 variable (2.8)	1) Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Alg:D2:C4:(A-APR.6) 2) Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x) and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.Alg:D4:C2:(A-REI.4a) 3) Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x-p)^2=q that has the same solutions. Derive the quadratic formula from this form. Alg:D4:C2:(A-REI.4b) 4) Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g. for x^2=49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a +bi for real numbers a and b. 5) Fun:D1:C2:(F-IF.4) For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximum and minimums; symmetries; end behavior; and periodicity.** 6) Fun:D1:C3:(F-IF.7c) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* (c.) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. 7) Fun:D1:C3:(F-IF.8a) Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. 8) Num:D3:C1:(N-CN.1) Know there is a complex number i such that i^2=-1 and every complex number has the form a+bi and a and b are real. 9) Num:D3:C3:(N-CN.7) Solve quadratic equations with real coefficients that have complex solutions. 10) Num:D3:C3:(N-CN.8)(+) (+) Extend polynomial identities to the complex numbers. *For example, rewrite x^2+4 as (x+2i)(x-2i).* 11) Num:D3:C3:(N-CN.9)(+) (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

4-5	Exponential, Logistic, and Logarithmic Functions	*Exponential and Logistic Functions (3.1) *Exponential and Logistic Modeling (3.2) *Logarithmic Functions and their graphs (3.3) *Properties of Logarithmic Functions (3.4) *Equation Solving and Modeling (3.5) *Mathematics of Finance (3.6)	Alg:D4:C4:(A-REI.11) Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Fun:D1:C3:(F-IF.7e) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* (e.) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Fun:D2:C2:(F-BF.5)(+) (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. Alg:D1:C2:(A-SSE.3) Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. _For example:_ _1.15^t can be rewritten as (1.15^(1/12))^(12t) approx. 1.012^(12t)_ _to reveal the approximate equivalent monthly interest rate if the_ _annual rate is 15%._ Alg:D3:C1:(A-CED.1) Create equations and inequalities in one variable and use them to solve problems. _Include equations arising from linear and quadratic functions, and simple rational and exponential functions._ Fun:D1:C3:(F-IF.7e) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* (e.) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Fun:D1:C3:(F-IF.8b) Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t, y=(0.97)^t, y=(1.01)^(12t), y=(1.2)^(t/10), and classify them as representing exponential growth or decay. Fun:D3:C1:(F-LE.4) For exponential models, express as a logarithm the solution to ab^(ct)=d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Fun:D2:C1:(F-BF.1b) Write a function that describes a relationship between two quantities.* (b.) Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model Fun:D2:C2:(F-BF.5)(+) (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. Fun:D3:C1:(F-LE.1a) Distinguish between situations that can be modeled with linear functions and with exponential functions. (a.) Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Fun:D3:C1:(F-LE.1b) Distinguish between situations that can be modeled with linear functions and with exponential functions. (b.) Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. Fun:D3:C1:(F-LE.1c) Distinguish between situations that can be modeled with linear functions and with exponential functions. (c.) Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. Fun:D3:C1:(F-LE.2) Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). Fun:D3:C1:(F-LE.3) Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function Fun:D3:C1:(F-LE.4) For exponential models, express as a logarithm the solution to ab^(ct)=d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Fun:D3:C2:(F-LE.5) Interpret the parameters in a linear, quadratic, or exponential function in terms of a context Num:D1:C1:(N-RN.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. _For example, we define 5 1/3 to be the cube root of 5 because we want (5 1/3) 3 = 5 (1/3)3 to hold, so (5 1/3) 3 must equal 5._ Num:D1:C1:(N-RN.2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.

8	Trigonometric Functions	*Angles and their measures (4.1) *Trigonometric functions of acute angles (4.2) *Trigonometry extended: the circular functions (4.3) *Graphs of sine and cosine: sinusoids (4.4) *Graphs of tangent, cotangent, secant, and cosecant (4.5) *Graphs of composite trigonometric functions (4.6) – (If time allows) *Inverse trigonometric functions (4.7) *Solving problems with trigonometry (4.8)	Trigonometric Functions F-TF: **Extend the domain of trigonometric functions using the unit circle.** 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. 2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counter-clockwise around the unit circle. 3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ & $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. 4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **Model periodic phenomena with trigonometric functions.** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. Similarity, Right Triangles, and Trigonometry G-SRT : **Define trigonometric ratios and solve problems involving right triangles.** 6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. 7. Explain and use the relationship between the sine and cosine of complementary angles. Interpreting Functions F-IF : **Analyze functions using different representations.** 8. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ (e.) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Interpreting Functions F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Trigonometric Functions F-TF 2. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. Trigonometric Functions F-TF **Extend the domain of trigonometric functions using the unit circle.** **Model periodic phenomena with trigonometric functions.** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ 6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. 7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★
4	Analytic Trigonometry	*Fundamental Identities (5.1) *Proving Trigonometric Identities (5.2) *Sum and difference identities (5.3) *Multiple angle identities (5.4) *Law of Sines (5.5) *Law of Cosines (5.6)	F-TF #1 – Prove the Pythagorean identity $\sin^2 x + \cos^2 x = 1$, and use it to find sin(x), cos(x) or tan(x) given sin(x), cos(x), or tan(x) and the quadrant. F-TF #2 – (+) Prove the addition and subtraction formula for sine, cosine, and tangent and use them to solve problems. G-SRT #10 – (+) Prove the Law of Sines and Law of Cosines and use them to solve problems. G-SRT #11 -- Understand and apply the laws of Sines and Cosines to find unknown measurements in right and non-right triangles. These are the only applicable CCSS for this unit, but other learning objectives include: 5.1 – Students will be able to use the trigonometric identities to simplify trigonometric expressions and solve trigonometric equations. 5.2 – Students will be able to prove trigonometric identities. 5.3 – Students will be able to apply the identities for the cosine, sine, and tangent of a difference or sum. 5.4 – Students will be able to apply the double-angle identities, power-reducing identities, and half-angle identities.

1-2 Honors only	Applications of Trigonometry	*Vectors in the Plane (6.1) *Dot Product of Vectors (6.2) *Parametric Equations and Motion (6.3) *Polar Coordinates (6.4) *Graphs of Polar Equations (6.5) *Trigonometric form of a complex number (6.6)	Num:D4:C1:(N-VM.1)(+) (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g. **v**, |**v**|, ||**v**||, v) Num:D4:C1:(N-VM.2)(+) (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. Num:D4:C1:(N-VM.3)(+) (+) Solve problems involving velocity and other quantities that can be represented by vectors. Num:D4:C2:(N-VM.4a)(+) (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically no the sum of the magnitudes. Num:D4:C2:(N-VM.4b)(+) (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. Num:D4:C2:(N-VM.4c)(+) (+) Understand vector subtraction **v-w** as **v+(-w)**, where **-w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. Num:D4:C2:(N-VM.5a)(+) (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g. as c(v1,v2)=(cv1,cv2). Num:D4:C2:(N-VM.5b)(+) (+) Compute the magnitude of a scalar multiple c**v** using ||c**v**||=|c|**v**. Compute the direction of c**v** knowing that when |c|**v** is not equal to zero, the direction of c**v** is either along **v** (for c>0) or against **v** (for c<0). Sections 6.3, 6.4 and 6.5 do not have matching CCSS; however, learning objectives include: *Eliminating the parameter *Graphing parametric relations *Using parametric equations to model motion *Converting from polar to rectangular form and vice versa *Plotting polar points *Graphing polar relations
1-2	Discrete Mathematics	*Basic Combinatorics (9.1) – If time allows *Binomial Theorem (9.2) *Sequences (9.4) *Series (9.5)	9.1 – Students will be able use the multiplication principle of counting, permutations, or combinations to count the number of ways that ta task can be done. 9.2 – Students will be able to find expand a power of a binomial using the binomial theorem or Pascal's triangle. 9.2 – Students will be able to find the coefficient of a given term of a binomial expansion. F-IF #3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F-BF #2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F-LQE #2 – Construct arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs A-SSE #4 – Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

	Other Topics as time permits	*De Moivre's theorem and nth roots (6.6) *Probability (9.3) *Partial Fractions (7.4) *More on limits (10.3) *Optimization (Set up and solve graphically)	Num:D3:C2:(N-CN.4)(+) (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. Num:D3:C2:(N-CN.5)(+) (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, (-1+sqr(3)i)^3=8 because (-1+sqr(3)i) has a modulus of 2 and an argument 120 degrees.* Num:D3:C2:(N-CN.6)(+) (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. Conditional Probability and the Rules of Probability S-CP **Understand independence and conditional probability and use them to interpret data.** 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not"). 2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. 3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. *4.* Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *5.* Recognize and explain the concepts of conditional probability and independence in every-day language and everyday situations. **Use the rules of probability to compute probabilities of compound events in a uniform probability model.** 6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also be-long to *A*, and interpret the answer in terms of the model. 7. Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model. 8. (+) Apply the general Multiplication Rule in a uniform probability model, *P*(*A* and *B*) = *P*(*A*)*P*(*B* | *A*) = *P*(*B*)*P*(*A* | *B*), and interpret the answer in terms of the model. 9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. Using Probability to Make Decisions S-MD **Calculate expected values and use them to solve problems.** 1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. 2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. 3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. 4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.